Journal of Applied Mechanics and Technical Physics, Vol. 41, No. 6, 2000

EFFECT OF THE CONDUCTANCE AND THICKNESS OF A CONDUCTING PLATE ON THE SIGNAL FROM A MATERIAL-VELOCITY INDUCTIVE TRANSDUCER

Yu. N. Zhugin and Yu. L. Levakova

UDC 532.593 + 538.5.54 + 621.3.08

The perturbation problem of the magnetic field of a constant-current turn located above a conducting plate set into motion by a plane shock wave with a rectangular profile is considered. It is shown that not only the velocity of the plate but also its dynamic conductivity can be determined on the basis of the electromotive force of induction recorded by means of the turn. For the case where the conductance of the plate is known for both the conducting half-space and for a plate whose thickness is comparable with the skin-layer thickness, approximate calculated dependences for the velocity of the plate are obtained. A comparison with experimental data and the clarification of the calculated dependences allows one to conclude that the approaches proposed can be used for determining the conductance of metals in shock-wave processes.

Introduction. Zhugin and Krupnikov [1] proposed an inductive transducer for registration of short-term processes occurring upon shock compression of condensed media. Based on it, the induction method of continuous registration of the velocity of condensed media in shock-wave processes was developed in [2]. The principle of operation of the inductive transducer is based on the oscillographic registration of the electromotive force (e.m.f.) of induction that arises in a constant-current coil (transducer) located above a conducting plate after the plate has been set into motion by a shock wave. The diameter of the plate and its conductance and thickness (0.2–0.3 mm for copper and aluminum) are quite large. Variation of the magnetic flux through the transducer loop is due to nonstationary eddy currents occurring in the thickness-variable surface of the metal plate (nonstationary skin effect). Zhugin and Krupnikov [2] showed experimentally that in the time intervals of the laboratory experiment we are interested in, copper and aluminum plates of sufficiently large thickness can be regarded as ideally conducting plates. At the same time, for lead and bismuth, which possess much lower conductances, this assumption is incorrect; therefore, in obtaining the corresponding calculated dependences, one should take into account the finiteness of their electric conduction.

In using plates whose thickness is smaller than the thickness of the nonstationary skin-layer, the effect of magnetic-field diffusion through the moving plate is observed experimentally; this effect is manifested in the decrease in the amplitude of the recorded signal. If the plate velocity is constant, the decrease is the more considerable the thinner the plate and the lower its conductance. An appreciable decrease in the signal amplitude is also observed in the case of the half-space whose conductance is much lower than that of aluminum and copper.

The electromagnetic method of measuring the parameters of shock-compressed media, which is close to the induction method in its physical principles, was considered by Fritz and Morgan [3]. In [3], a formula that allows one to measure the material velocity and conductances of metal films of negligibly small thickness in a

Zababakhin Institute of Technical Physics, Snezhinsk 456770. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 6, pp. 199–209, November–December, 2000. Original article submitted November 10, 1999; revision submitted January 25, 2000.

 0021-8944/00/4106-1136 $25.00

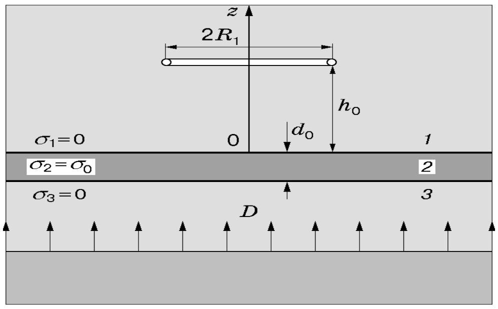

Fig. 1. Computational scheme.

layered dielectric–film–dielectric system was also obtained by the image method. However, the experimental data concerning the use of this system for measuring the material velocity and conductance were not given in [3].

The goal of the present study is to obtain (with the use of an approach different from that used in [3]) calculated dependences for the e.m.f. of induction that take into account the finiteness of the conductance of metal plates of different thickness. As is noted in [4], similar problems cannot be solved without cumbersome numerical computations. This circumstance makes it necessary to use approximate approaches allowing for the basic specific features of the interaction between the moving conductors with electromagnetic fields, which is convenient for the solution of applied problems.

1. Formulation of the Problem. A turn of radius R_1 with a negligibly small wire cross section is connected to a stabilized constant-current source and located in condensed dielectric medium 1 at height h_0 over a conducting plate of thickness d_0 (medium 2) with conductance σ_0; a stationary magnetic field is established in the entire pace (Fig. 1). The magnetic permeability of the media is $\mu\mu_0$ ($\mu = 1$; μ_0 is the magnetic permeability of vacuum).

A plane shock wave whose front is parallel to the interface of the media propagates upward over medium 3 with velocity D. In practice, the interaction between the moving shock-compressed dielectric and the magnetic field does not occur [2, 5] if the magnetic Reynolds number is $\mathrm{Re}_m = \mu_0\sigma u R_1 \ll 1$; for the material velocity $u = 5 \times 10^3$ m/sec, this is equivalent to the condition $\sigma \ll 10^4$ $(\Omega \cdot \mathrm{m})^{-1}$. For the majority of condensed dielectrics, the latter condition is satisfied in a broad range of shock-compression pressures.

The problem of shock-wave propagation in an unlimited conductor in the presence of a magnetic field was considered by Burgers [4] and Zababakhin and Nechaev [6]. It was shown that if a metal possesses an infinite conductance, the magnetic field in front of the shock wave remains unchanged. In the case of finite conductance, the shock wave influences a conductor before it in the adjacent layer of thickness $l \approx 1/(\mu_0\sigma D)$. The width of the electromagnetic wave is 2.5 μm in copper ($D = 5 \cdot 10^3$ m/sec), 4 μm in aluminum, and 30 μm in lead. It is evident that, for these metals, the turn practically does not react to the motion of the shock wave over the conductor until it reaches the interface of media 1 and 2.

It is known that the characteristic time for which the magnetic field damps or diffuses is $t \approx \mu_0\sigma\zeta^2$ (ζ is the distance on which the magnetic field changes noticeably) [5]. In this case, the change in the field begins with the conductor surface. Upon decay of the discontinuity at the boundary of mediums 1 and 2, the reflected wave in the conductor (for example, in copper) propagates over a thickness approximately equal to $D_1 t$ for t and over the thickness $D_1 t \geqslant \zeta$ for the time $t \geqslant \tau = 1/(\mu_0\sigma D_1^2) \simeq 5 \times 10^{-4}$ μsec.

We assume that at the moment $t = 0$ the entire plate is compressed by δ times and acquires, in the direction of the turn, the material velocity u, which is due to the shock wave's reaching the interface of media 1 and 2. In the experiments described in [2], the behavior of metals at pressures of 10–20 GPa for $\delta \simeq 1.1$–1.2 was studied by means of an inductive transducer. One can ignore the magnetic-field distortions caused by the effects of magnetic field line "freezing-in" into a conductor with high conductance, i.e., one can consider that the magnetic field in a compressed metal for $t = +0$ is determined by the initial magnetic field of the turn with a constant current.

To solve the posed problem, a cylindrical coordinate system connected to the conductor surface is used (Fig. 1). In this coordinate system, the constant-current turn approaches the interface of media 1 and 2 with velocity u, thereby predetermining the variability of the magnetic field on the conductor surface and, consequently, the occurrence of nonstationary concentric eddy currents in the surface layer. They change the field in medium 1 and cause the appearance of the e.m.f. of induction that depends on the velocity of the conductor, its conductance, and the plate thickness, which is quite small.

2. The Helmholtz Equation for the Vector Potential of an Electromagnetic Field. *Quasistationary Field.* In the case of variable fields for immovable, homogeneous, and isotropic media with ε, μ, and σ constant over the volume, the Helmholtz equation has the form

$$\Delta \boldsymbol{A} - \varepsilon_0\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \mu_0\mu\sigma \frac{\partial \boldsymbol{A}}{\partial t} = -\mu_0\mu \boldsymbol{j}_{(e)},$$

where $\boldsymbol{A}$ is the vector potential of the electromagnetic field, $\boldsymbol{j}_{(e)}$ is the density of extraneous currents, t is the time, ε and μ are the dielectric and magnetic permeabilities of the medium, respectively, and σ is its conductance, and ε_0 and μ_0 are the dielectric and magnetic permeabilities of vacuum, respectively [7]. The transducer field in the dielectric is considered quasistationary, which implies that the wave processes in media 1 and 3 can be ignored. This simplification is justified if the condition $L \ll ct$ is satisfied (L and t are the characteristic dimension and time of the system, respectively, and c is the velocity of light). In a conducting medium, we assume that $\mu = 1$ and consider only the processes that are due to the presence of conduction, i.e., as in dielectrics, the displacement currents are ignored.

Thus, we have

$$\Delta \boldsymbol{A} = -\mu_0 \boldsymbol{j}_{(e)} \tag{2.1}$$

outside the conductor ($\sigma = 0$) and

$$\Delta \boldsymbol{A} = \mu_0\sigma \frac{\partial \boldsymbol{A}}{\partial t} \tag{2.2}$$

for the conductor ($\boldsymbol{j}_{(e)} = 0$).

Solution of the Helmholtz Equation for Region 1 (*Dielectric*). We use the cylindrical coordinate system (ρ, φ, z) whose z axis is related to the conductor, directed normally to it, and coincides with the turn axis. The coordinate origin is placed on the conductor surface (Fig. 1). We consider that the diameter of the turn cross section is small compared with the turn radius R_1; in other words, the current flows along the line with coordinates $\rho = R_1$ and $z = h$. Using the Dirac delta-function, we write the current-density expression in the form

$$j_{(e)} = I_0\delta(z - h)\delta(\rho - R_1). \tag{2.3}$$

By virtue of the axial symmetry, the vector-potential problem also has only the component φ and does not depend on the angle of φ, i.e., $A = A_\varphi$. Then, in cylindrical coordinates, Eq. (2.1) takes the form (see, e.g., [7])

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial A}{\partial\rho}\Big) + \frac{\partial^2 A}{\partial z^2} - \frac{A}{\rho^2} = -\mu_0 j_{(e)}. \tag{2.4}$$

According to the technique from [7, 8], Eq. (2.4) can be solved by using the integral Fourier–Bessel transform with a kernel in the form of a first-order Bessel function. The transformation formula has the form

$$A^* = \int_0^\infty \rho J_1(\lambda\rho) A(\rho, z)\, d\rho, \tag{2.5}$$

where λ is the transformation parameter. Applying the transformation (2.5) to both sides of Eq. (2.4), we obtain

$$\frac{d^2 A^*}{dz^2} - \lambda^2 A^* = -\mu_0 j^*_{(e)},$$

where A^* is a function of coordinate z and $j^*_{(e)}$ is the transformed current density. The general solution of this equation can be presented in the form

$$A^* = \frac{\mu_0}{2\lambda}\left[\mathrm{e}^{\lambda z}\left(B - \int_0^z j^*_{(e)}\mathrm{e}^{-\lambda\xi}\,d\xi\right) + \mathrm{e}^{-\lambda z}\left(C + \int_0^z j^*_{(e)}\mathrm{e}^{\lambda\xi}\,d\xi\right)\right], \tag{2.6}$$

where ξ is the integration variable along the direction of z and B and C are the z-independent quantities which can be determined from the boundary conditions.

As $z \to \infty$, the field should be limited. This is possible under the condition

$$B = \int_0^\infty j^*_{(e)}\mathrm{e}^{-\lambda\xi}\,d\xi.$$

Taking into account relation (2.3), we obtain

$$j^*_{(e)} = \int_0^\infty \rho J_1(\lambda\rho) j_{(e)}(\rho, z)\,d\rho = I_0 \int_0^\infty \rho J_1(\lambda\rho)\delta(\rho - R_1)\delta(z - h)\,d\rho = I_0 R_1 J_1(\lambda R_1)\delta(z - h),$$

$$B = I_0 R_1 J_1(\lambda R_1) \int_0^\infty \delta(\xi - h)\mathrm{e}^{-\lambda\xi}\,d\xi = I_0 R_1 J_1(\lambda R_1)\mathrm{e}^{-\lambda h}.$$

Sobolev and Shkarlet showed in [8] that the desired quantity $C = B\varphi_1$, and with allowance for

$$\int_0^z j^*_{(e)}(\lambda, \xi)\mathrm{e}^{-\lambda\xi}\,d\xi = \begin{cases} 0, & z < h, \\ I_0 R_1 J_1(\lambda R_1)\mathrm{e}^{-\lambda h}, & z > h, \end{cases}$$

$$\int_0^z j^*_{(e)}(\lambda, \xi)\mathrm{e}^{+\lambda\xi}\,d\xi = \begin{cases} 0, & z > h, \\ I_0 R_1 J_1(\lambda R_1)\mathrm{e}^{\lambda h}, & z < h, \end{cases}$$

expression (2.6) takes the form

$$A^*_1 = \mu_0(\mathrm{e}^{-\lambda|z-h|} + \varphi_1\mathrm{e}^{-\lambda(z+h)}) I_0 R_1 J_1(\lambda R_1)/(2\lambda). \tag{2.7}$$

It is clear that the problem of field determination in region 1 is reduced to searching for the form of the function φ_1.

To determine φ_1, the known boundary conditions for the vector potential of a magnetic field and its derivative with respect to z, which remain true for corresponding quantities obtained by various transformations, are used in [7, 8].

In region 1, for $z < h$ we have

$$A^*_1 = K_\lambda(\mathrm{e}^{-\lambda(h-z)} + \varphi_1\mathrm{e}^{-\lambda(h+z)}), \tag{2.8}$$

where $K_\lambda = \mu_0 I_0 R_1 J_1(\lambda R_1)/(2\lambda)$ and $h = h_0 - ut$. Then,

$$A^*_1 = K_\lambda(\mathrm{e}^{-\lambda(h_0-z)}\mathrm{e}^{\lambda ut} + \mathrm{e}^{-\lambda(h_0+z)}\Phi_1(t)), \quad \frac{dA^*_1}{dz} = \lambda K_\lambda(\mathrm{e}^{-\lambda(h_0-z)}\mathrm{e}^{\lambda ut} - \mathrm{e}^{-\lambda(h_0+z)}\Phi_1(t)), \tag{2.9}$$

where $\Phi_1 = \varphi_1\mathrm{e}^{\lambda ut}$.

Applying the Laplace transform [9] to both sides of Eqs. (2.9), we obtain the equations

$$\begin{aligned} L[A^*_1] &= \frac{K_\lambda\mathrm{e}^{-\lambda(h_0-z)}}{s - \lambda u} + K_\lambda\mathrm{e}^{-\lambda(h_0+z)}L[\Phi_1(t)], \\ L\left[\frac{dA^*_1}{dz}\right] &= \frac{\lambda K_\lambda\mathrm{e}^{-\lambda(h_0-z)}}{s - \lambda u} - \lambda K_\lambda\mathrm{e}^{-\lambda(h_0+z)}L[\Phi_1(t)]. \end{aligned} \tag{2.10}$$

Solution of the Helmholtz Equation for Region 2 (Conductor). In cylindrical coordinates, Eq. (2.2) has the form

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\,\frac{\partial A_2}{\partial\rho}\Big)+\frac{\partial^2 A_2}{\partial z^2}-\frac{A_2}{\rho^2}=\mu_0\sigma\,\frac{\partial A_2}{\partial t}. \tag{2.11}$$

Applying the Fourier–Bessel transform to Eq. (2.11), we obtain

$$\frac{\partial^2 A_2^*}{\partial z^2}-\lambda^2 A_2^*=\mu_0\sigma\,\frac{\partial A_2^*}{\partial t}. \tag{2.12}$$

Applying the Laplace transform to (2.12), we have

$$L\Big[\frac{d^2 A_2^*}{dz^2}\Big]-\lambda^2 L[A_2^*]=\frac{1}{a^2}\,(sL(A_2^*)-A_{20}^*), \tag{2.13}$$

where s is the Laplace-transform parameter, A_{20}^* is the initial magnitude of the transformed vector potential A_2^*, and $a^2=1/(\mu_0\sigma)$. After simple transformations of Eq. (2.13), with allowance for expressions (2.7), for the turn field in a free space ($\varphi_1=0$) we obtain

$$L\Big[\frac{d^2 A_2^*}{dz^2}\Big]-\beta^2 L[A_2^*]=-\frac{K_\lambda}{a^2}\,\mathrm{e}^{-\lambda(h_0-z)}, \tag{2.14}$$

where $\beta^2=(\lambda^2a^2+s)/a^2$.

It follows from Eq. (2.14) that

$$L[A_2^*]=A\mathrm{e}^{\beta z}+B\mathrm{e}^{-\beta z}+\frac{K_\lambda\mathrm{e}^{-\lambda(h_0-z)}}{s},\qquad L\Big[\frac{dA_2^*}{dz^2}\Big]=A\beta\mathrm{e}^{\beta z}-B\beta\mathrm{e}^{-\beta z}+\frac{\lambda K_\lambda\mathrm{e}^{-\lambda(h_0-z)}}{s}. \tag{2.15}$$

Solution of the Helmholtz Equation for Region 3 (Dielectric). In region 3 ($j_{(e)}=0$ and $\sigma=0$), the Helmholtz equation is reduced to the form

$$L\Big[\frac{d^2 A_3^*}{dz^2}\Big]-\lambda^2 L[A_3^*]=0$$

by means of the Fourier–Bessel and Laplace transforms and has the solution $L[A_3^*]=D\mathrm{e}^{\lambda z}+E\mathrm{e}^{-\lambda z}$.

As $z\to-\infty$, the field is bounded; therefore, the coefficient E should be zero. Thus,

$$L[A_3^*]=D\mathrm{e}^{-\lambda z},\qquad L\Big[\frac{dA_3^*}{dz}\Big]=D\lambda\mathrm{e}^{\lambda z}. \tag{2.16}$$

3. Field of the Turn Positioned above the Conducting Half-Space and a Plate of Finite Thickness Which Are Driven by a Shock Wave. Using relations (2.10), (2.15), and (2.16), we require the fulfillment of the boundary conditions for the transformed vector potentials and their derivatives for $z=0$ and $z=-d$ [7, 8].

The system of equations takes the following form:

$$A+B-K_\lambda\mathrm{e}^{-\lambda h_0}L[\Phi_1]=K_\lambda\mathrm{e}^{-\lambda h_0}\Big(\frac{1}{s-\lambda u}-\frac{1}{s}\Big),$$

$$A-B+\frac{K_\lambda\mathrm{e}^{-\lambda h_0}}{\alpha}\,L[\Phi_1]=\frac{K_\lambda\mathrm{e}^{-\lambda h_0}}{\alpha}\Big(\frac{1}{s-\lambda u}-\frac{1}{s}\Big), \tag{3.1}$$

$$A\mathrm{e}^{-\beta d}+B\mathrm{e}^{\beta d}+K_\lambda\mathrm{e}^{-\lambda h_0-\lambda d}/s=D\mathrm{e}^{-\lambda d},\quad A\alpha\mathrm{e}^{-\beta d}-B\alpha\mathrm{e}^{\beta d}+K_\lambda\mathrm{e}^{-\lambda h_0-\lambda d}/s=D\mathrm{e}^{-\lambda d}.$$

Here $\alpha=\beta/\lambda=\sqrt{s+\lambda^2a^2}/(\lambda a)$. Solving system (3.1), we find that

$$L[\Phi_1]=\frac{1-\alpha}{1+\alpha}\Big(\frac{1}{s-\lambda u}-\frac{1}{s}\Big)\Big[1+\frac{4\alpha\mathrm{e}^{-2\beta d}}{(1-\alpha)^2\mathrm{e}^{-2\beta d}-(1+\alpha)^2}\Big]. \tag{3.2}$$

It is easy to show that the solution is correct, because we have $L[\Phi_1]=L[\varphi_1]=0$ for $d=0$, i.e., the perturbation of the initial field of the turn is absent.

Conducting Half-Space ($d\to\infty$). It follows from (3.2) that as $d\to\infty$,

$$L[\Phi_1] = \frac{1-\alpha}{1+\alpha}\Big(\frac{1}{s-\lambda u} - \frac{1}{s}\Big). \tag{3.3}$$

Ideal-Conduction Approximation. We consider the simplest case where the electric conduction is quite high. One can let the quantity $a = \sqrt{1/(\mu_0\sigma)}$ tend to zero, and α to infinity [see formula (3.3)]. Then, we have $L[\Phi_1] = 1/s - 1/(s-\lambda u)$ or $\Phi_1 = 1 - \mathrm{e}^{\lambda ut}$ (after the inverse Laplace transform [9]). Taking into account that $\Phi_1 = \varphi_1 \mathrm{e}^{\lambda ut}$, we obtain

$$\varphi_1 = -1 + \mathrm{e}^{-\lambda ut}. \tag{3.4}$$

Using the functions φ_1, one can find a formula for the e.m.f. of induction occurring in the turn ($z = h$ and $\rho = R_1$) after the surface of the ideally conducting half-space ($\sigma \to \infty$) is set into motion by a shock wave.

Applying the inverse Fourier–Bessel transform, we find the true value of the field in region 1:

$$A_1 = \int\limits_0^\infty A_1^*(\lambda, R_1) J_1(\lambda\rho)\lambda\, d\lambda.$$

With allowance for (2.8), we obtain

$$A_1 = \frac{\mu_0 R_1 I_0}{2}\Big[\int\limits_0^\infty J_1(\lambda R_1)J_1(\lambda\rho)\mathrm{e}^{-\lambda|z-h|}\, d\lambda + \int\limits_0^\infty J_1(\lambda R_1)J_1(\lambda\rho)\varphi_1\mathrm{e}^{-\lambda|z+h|}\, d\lambda\Big].$$

The e.m.f. of induction has the form

$$E(t) = -\frac{d}{dt}\oint A_1\, dl = -\mu_0 I_0 \pi R_1^2 \int\limits_0^\infty J_1^2(\lambda R_1)\Big(\frac{d\varphi_1}{dt} + 2\lambda u\varphi_1\Big)\mathrm{e}^{-2\lambda h}\, d\lambda \tag{3.5}$$

(integration is performed along the turn loop). Substituting (3.4) into (3.5) and taking into account that

$$\int\limits_0^\infty J_1^2(\lambda R_1)\mathrm{e}^{-m\lambda}\lambda\, d\lambda = \frac{m}{\pi R_1^3}\,\frac{k}{4}\Big[\frac{2-k^2}{1-k^2}E(k^2) - 2K(k^2)\Big]$$

[$k^2 = 1/[1 + (m/(2R_1))^2\,]$ and $E(k^2)$ and $K(k^2)$ are the full elliptic integrals of the first and second kinds], one can be convinced that expression (3.5) coincides with the expression for the e.m.f. of induction obtained in [2] by the image method.

Half-Space with Finite Conductance. Now we let the quantities proportional to $a^2 = 1/(\mu_0\sigma)$ tend to zero and leave the quantities proportional to $a = 1/\sqrt{\mu_0\sigma}$. We obtain

$$L[\Phi_1] = \frac{1}{s} - \frac{1}{s-\lambda u} + \frac{2a\lambda}{(s-\lambda u)(a\lambda + \sqrt{s})} - \frac{2a\lambda}{s(a\lambda+\sqrt{s})}. \tag{3.6}$$

Applying the inverse Laplace transform and the relation $\varphi_1 = \Phi_1\mathrm{e}^{-\lambda ut}$ to (3.6), we have

$$\varphi_1 = -1 + \mathrm{e}^{-\lambda ut} + \frac{2a\lambda}{\sqrt{\lambda u}}\,\mathrm{erf}\sqrt{\lambda ut} - \frac{4a\lambda\sqrt{t}}{\sqrt{\pi}}\,\mathrm{e}^{-\lambda ut}. \tag{3.7}$$

According to (3.5), we find that for the half-space of finite conductance, the e.m.f. is

$$E_\sigma = E_{\sigma\to\infty} - 4I_0R_1^2u\sqrt{\frac{\pi\mu_0 t}{\sigma}}\int\limits_0^\infty J_1^2(\lambda R_1)\lambda^2\mathrm{e}^{-2\lambda h}\Big\{\sqrt{\pi}\,\frac{\mathrm{erf}\sqrt{\lambda ut}}{\sqrt{\lambda ut}} - \mathrm{e}^{-\lambda ut}\Big\}\, d\lambda. \tag{3.8}$$

By analogy with the case of ideal conduction, the dependence (3.8) can be represented as $E_\sigma = I_0\alpha_\sigma u$. To estimate quantitatively the dependence $\alpha_\sigma(t)$, with the material velocity known, one can use the initial value of the conductance of the metal. More exact data can be obtained by measuring the conductance of the metal under close loading conditions by known methods [10, 11], in particular, by the method of thin plates.

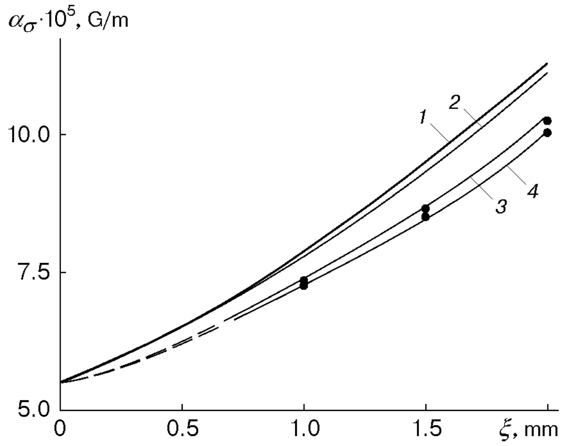

Fig. 2

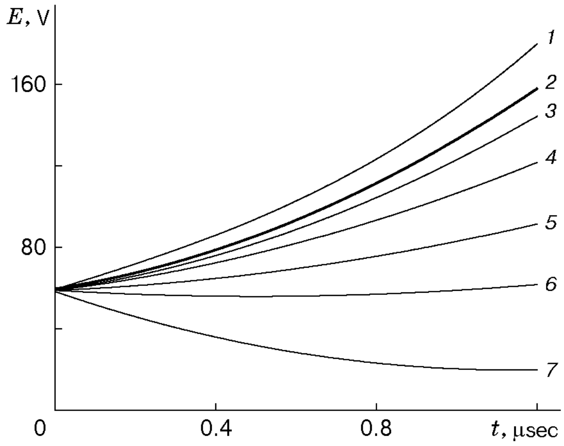

Fig. 3

Fig. 2. Calculated dependences $\alpha_\sigma(\xi)$ for the ideally conducting half-space (curve 1) and the half-space from copper (curve 2) and the experimental dependences for lead plates (curves 3 and 4).

Fig. 3. Calculated dependences $E(t)$ for plates of different thickness d_0 with finite conductance σ as $\sigma \to \infty$ (1) and $d_0 \to \infty$ (2) and for 0.20 (3), 0.10 (4), 0.05 (5), 0.03 (6), 0.01 mm (7).

Comparison with Experiment. In [2], for a comparative analysis, lead, whose initial specific conductivity is a factor of 10 smaller than that of copper was used. Two series of experiments were performed. In the first series, a plane shock wave with a rectangular structure was first introduced from the aluminum screen (thickness 10 mm) of an explosive device (diameter 120 mm) into a lead sample and then into a polymethylmethacrylate sample. The thicknesses of the samples were 5 and 6 mm, respectively, and their diameter was 100 mm. The signal was recorded by a transducer, i.e., by an induction coil consisting of 8 coils of isolated copper wire of diameter 1 mm and average radius $R_N \simeq 16$ mm; the thickness and width of the shells were 2.5 and 5 mm, respectively. In this case, the e.m.f. is increased by a factor of N^2 [2].

The second series of experiments differed from the first series in that a copper plate of diameter 100 mm and thickness 0.3 mm was placed on the lead–dielectric interface; this plate quickly acquired the velocity of this interface, which made it possible to determine the material velocity of the lead–polymethylmethacrylate interface [2]. Two types of explosive devices, I and II, with known shock-wave parameters behind its front in the aluminum screen were used in experiments, which ensured initial pressures of 41 and 79 GPa, respectively, in the lead sample and 10 and 20 GPa, respectively, in the polymethylmethacrylate sample.

In Fig. 2, the calculated dependence $\alpha_\sigma(\xi)$ for the ideal conductor (curve 1) and the calculation results obtained by formula (3.8) for copper (curve 2) are compared with the experimental data for lead (curves 3 and 4 for explosive devices II and I, respectively). For the displacement $\xi = 2$ mm, the value of α_σ for the ideal conductor is 1% greater than that of α_σ for copper. The dynamic values of the conductance, which are $4.3 \cdot 10^6$ and $4.1 \cdot 10^6$ $(\Omega \cdot \mathrm{m})^{-1}$ for curves 3 and 4, were estimated by means of relations (3.8) for experiments with lead. The degree of correspondence of the calculated values of the conductance to the experimental curves is shown by filled circles in Fig. 2.

Plate of Finite Thickness. We pass to an approximate analysis of the part of the solution (3.2) that depends on the thickness of the metal plate:

$$L[\Delta\Phi_1] = \frac{1-\alpha}{1+\alpha}\Big(\frac{1}{s-\lambda u} - \frac{1}{s}\Big)\frac{4\alpha \mathrm{e}^{-2\beta d}}{(1-\alpha)^2\mathrm{e}^{-2\beta d} - (1+\alpha)^2}. \qquad (3.9)$$

We assume that $(1-\alpha)/(1+\alpha) \simeq -1$. On the right side of expression (3.9) containing the terms with the conducting-plate thickness d, we let only the quantities proportional to $a^2 = 1/(\mu_0\sigma)$ tend to zero and keep the quantities containing $a = 1/\sqrt{\mu_0\sigma}$. Here relation (3.9) takes the form

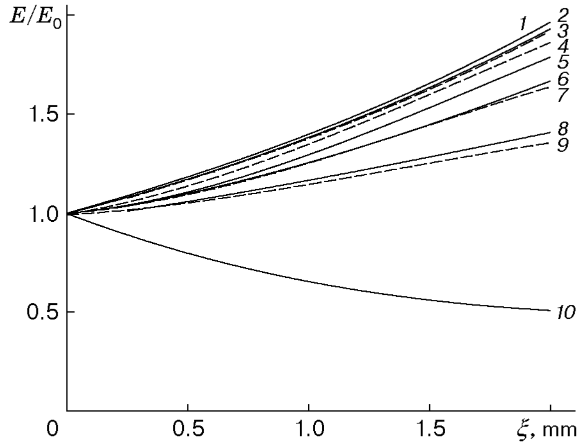

Fig. 4. Smoothed experimental dependences $E/E_0(\xi)$ for aluminum and copper plates of different thickness d_0 [mm]: curve 1 refers to $\sigma \to \infty$, and curves 2–10 to $d_0 = 0.20$ (Cu), 0.30 (Al), 0.10 (Cu), 0.10 (Al), 0.05 (Al), 0.03 (Cu), 0.03 (Al), 0.015 (Cu), and 0.01 (Al), respectively.

$$L[\Delta\varphi_d] = \frac{-4a\lambda \exp(-2d\sqrt{s}/a)/(s-\lambda u) + 4a\lambda \exp(-2d\sqrt{s}/a)/s}{\sqrt{s}\,(\exp(-2d\sqrt{s}/a) - 1) - 2a\lambda(\exp(-2d\sqrt{s}/a) + 1)}. \tag{3.10}$$

The inverse Laplace transform for $L[\Delta\varphi_d]$ (3.10) leads to a convolution-type integral Volterra equation of the first kind [9]:

$$\int_0^t \Delta\varphi_d(\tau)\exp(\lambda u\tau)\Big\{\frac{\exp(-d^2/[a^2(t-\tau)]) - 1}{\sqrt{\pi}(t-\tau)} - 2a\lambda\Big[1 + \operatorname{erfc}\Big(\frac{d}{a\sqrt{t-\tau}}\Big)\Big]\Big\}\,d\tau$$

$$= 4a\lambda \int_0^t \operatorname{erfc}\frac{d}{a\sqrt{\tau}}\,d\tau - 2a\lambda \int_0^t \exp(\lambda u\tau)\Big\{\exp\Big(-\frac{2d}{a}\sqrt{\lambda u}\Big)\operatorname{erfc}\Big(\frac{d}{a\sqrt{\tau}} - \sqrt{\lambda u\tau}\Big)$$

$$+ \exp\Big(\frac{2d}{a}\sqrt{\lambda u}\Big)\operatorname{erfc}\Big(\frac{d}{a\sqrt{t}} + \sqrt{\lambda u\tau}\Big)\Big\}\,d\tau. \tag{3.11}$$

The functions $\Delta\varphi_d(t)$ determined for plates of different thickness whose conductance is close to that of lead were calculated numerically by Ya. K. Khisamdinov and A. A. Trusnikov. In the calculations, the integrals on the left and right sides of the integral equation (3.11) are replaced by the sums, which makes it possible to pass to a system of algebraic equations [12]. Figure 3 shows an example of the dependences $E(t)$ that correspond to φ_1 relative to (3.7) with the additional terms $\Delta\varphi_d$ determined from (3.11). The initial parameters are as follows: $\sigma = 5.865 \cdot 10^6$ $(\Omega\cdot\text{m})^{-1}$, $u = 2.7\cdot 10^3$ m/sec, $I_0 = 500$ A, $N = 8$, $R_N = 16\cdot 10^{-3}$ m, $h_0 = 8.25 \cdot 10^{-3}$ m, and $\delta = 1.2$.

Experiments with Copper and Aluminum Plates of Finite Thickness. If the thickness of the plate reaches values smaller than the thickness of the surface current layer in the metal, the effect of magnetic-field diffusion through it begins to be manifested, which leads to a decrease in the recorded signal. The influence of this effect on the signal was studied in a number of experiments with the use of copper and aluminum as an example [2]. The thickness of the plate (foil) whose diameter is not smaller than 100 mm changed from 0.01 to 0.3 mm. The dielectric medium in which the foil (polymethylmethacrylate) was placed was subjected to loading by a plane shock wave with a rectangular profile. It was found that, for a pressure of $P \simeq 20$ GPa in the dielectric, the signal practically does not depend on the foil thickness and it is not smaller than 0.1 mm for copper and 0.2 mm for aluminum. For a pressure of $P \simeq 60$ GPa in the dielectric, the critical thickness of the copper and aluminum foils is 0.2 and 0.3 mm, respectively.

Figure 4 shows the experimental dependence E/E_0 on the displacement $\xi = ut$ for copper and aluminum foils of different thickness d_0. In the experiments, polymethylmethacrylate was used as a dielectric, and the loading parameters were as follows: $u = 2.45 \cdot 10^{-3}$ m/sec and $P = 18.5$ GPa. The induction coil in which

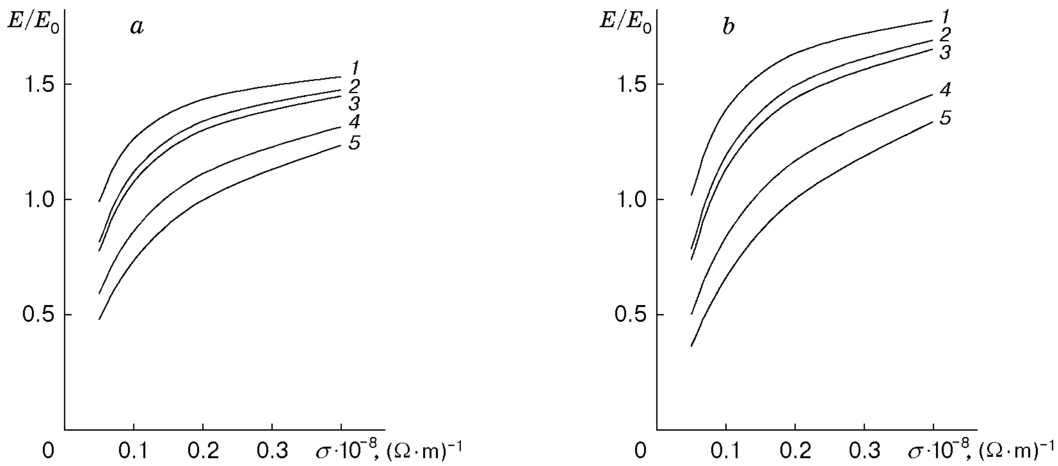

Fig. 5. Calculated dependences $E/E_0(\sigma)$ for $t = 0.6$ (a) and 0.8 μsec (b) for aluminum and copper plates of different thickness d_0 [mm]: curves 1–5 refer to $d_0 = 0.05$ (Al), 0.03 (Cu), 0.03 (Al), 0 . 015 (Cu), and 0.01 (Al), respectively.

the number of turns was $N = 8$ and whose average radius was $R_N = 15.85 \cdot 10^{-3}$ m was used as a transducer. The distance from the conducting surface to the average cross section of the coil was $h_0 = 8.16 \cdot 10^{-3}$ m and the current was $I_0 = 500$ A. It follows from a comparative analysis of Figs. 3 and 4 that the calculated and experimental dependences E/E_0 on the value of ξ are qualitatively similar.

We also attempted to determine the dynamic values of the conductance of copper and aluminum for the loading conditions indicated above. With the use of the values of φ_1 calculated from formulas (3.7) and $\Delta\varphi_d$ from (3.11) for $t = 0.6$ and 0.8 μsec, the calculated dependences E/E_0 on the conductance σ (Fig. 5a and b, respectively) were obtained for experiments with foils of different thickness d_0. In comparison with the initial value, the dynamic values of the conductance decreased by approximately a factor of 1.9 for $d_0 = 0.015$ and 0.03 mm for copper and a factor of 2.1 for $d_0 = 0.03$ and 0.05 mm and a factor of 6 for $d_0 = 0.01$ mm for aluminum.

Conclusions. The calculated dependences given in the work for determination of the velocity of metal plates of sufficiently large thickness (in the electromagnetic sense) with allowance for the finiteness of their conductance contributes to the development of the induction method of measuring the material velocities of condensed substances [2].

The possibility of contact-free conductance measurements of metals in shock-wave processes has been shown. At the given stage of studies, it is difficult to give a correct interpretation of the calculated and experimental results with the use of the estimates of the dynamic values of the conductance. This will become possible after the electromagnetic model is improved, the error of conductance measurement is estimated, and the range of applicability of the calculated dependences is found.

REFERENCES

1. Yu. N. Zhugin and K. K. Krupnikov, USSR Inventor's Certificate No. 468150, G 01p3/42, "Inductive transducer for recording of short-term processes," No. 1861600; Applied 12.25.72, Published 04.25.75; Bull. No 15, Priority 12.25.72.
2. Yu. N. Zhugin and K. K. Krupnikov, "Induction method of continuous recording of the velocity of a condensed medium in shock-wave processes," *Prikl. Mekh. Tekh. Fiz.*, No. 1, 102–108 (1983).
3. J. N. Fritz and J. A. Morgan, "An electromagnetic technique for measuring material velocity," *Rev. Sci. Instr.*, **44**, No. 2, 215–221 (1973).
4. Zh. M. Burgers, "Penetration of the shock wave into a magnetic field," in: Magnetic Hydrodynamics (Proc. of the Symposium), Atomizdat, Moscow (1958).

5. Y. A. Shercliff, *A Textbook of Magnetohydrodynamics*, Pergamon Press, Oxford–New York (1965).
6. E. I. Zababakhin and M. N. Nechaev, "Shock waves of the field and their cumulation," *Zh. Éksp. Teor. Fiz.*, **33**, No. 2, 442–50 (1957).
7. G. A. Grinberg, *Some Problems of the Mathematical Theory of Electrical and Magnetic Phenomena* [in Russian], Izd. Akad. Nauk SSSR, Moscow–Leningrad (1948).
8. V. S. Sobolev and Yu. M. Sharlet, *Superposed and Screen Transducers* [in Russian], Nauka, Novosibirsk (1967).
9. V. A. Ditkin and A. P. Prudnikov, *Manual of Operational Calculations* [in Russian], Vysshaya Shkola, Moscow (1965).
10. R. Keeler, "Electrical conduction of condensed media at high pressures," in: P. Caldirola and H. Knoepfel (eds.), *Physics of High Energy Density* Academic Press, New York–London (1971).
11. A. I. Goncharov and V. N. Rodionov, "Electrical resistance of copper and aluminum under shock-wave loading," in: *Lavrent'ev Readings in Mathematics, Mechanics, and Physics*, Abstracts of the II All-Union Conf. (Kiev, Sept. 9–11, 1985), Naukova Dumka, Kiev (1985), pp. 72–73.
12. M. L. Krasnov, A. I. Kiselev, and G. I. Makarenko, *Integral Equations* [in Russian], Nauka, Moscow (1968).